

采用二阶迎风格式的 P ISO 算法研究

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【摘要】在 P ISO 算法基础上,发展了一种新的二阶迎风格式以提高计算精度又保证算法的稳定性。新格式应用于液体火箭发动机燃烧流动过程计算,其数值稳定性很好。

主题词: 喷雾燃烧 P ISO 算法 流动 计算

分类号: O 241

1 引言

在燃烧流动过程的数值模拟中应用最多的是 S M P L E 算法,但由于 S M P L E 算法采用压力修正方程在迭代计算中改进速度场,因而不能计算非定常流动。P ISO 算法继承了 S M P L E 算法的基本思想,但由连续方程和动量方程的离散形式导出一准确的压力方程,而不是一个近似的压力修正方程^[1,2]。此外, P ISO 算法采用一步隐格式预测、两步显格式校正迭代方案,使算法具有时间二阶精度,可以用于非定常流动计算^[3]。在 S M P L E 和 P ISO 算法中常用差分格式有迎风格式、混合格式、指数格式等,这些格式都是空间一阶精度,国外期刊已拒绝发表采用这类格式的文章^[4]。本文构造了二阶迎风格式,并与 P ISO 算法结合在一起应用于液体火箭发动机燃烧过程数值计算,检验了算法的可靠性。

2 P ISO 算法

2.1 流动控制方程

为了方便处理复杂流动边界条件,控制方程进行了坐标变换。燃烧室内流动控制方程在圆柱坐标系下变换到任意曲线坐标系下,其通用形式为:

$$\begin{aligned} & \frac{\partial \varphi}{\partial t} + \frac{1}{rJ} \frac{\partial}{\partial \xi} (rJ \rho U \varphi) + \frac{1}{rJ} \frac{\partial}{\partial \eta} (rJ \rho V \varphi) \\ &= \frac{1}{rJ} \frac{\partial}{\partial \xi} \left(\frac{rJ \varphi}{J} \alpha \frac{\partial \varphi}{\partial \xi} \right) + \frac{1}{rJ} \frac{\partial}{\partial \eta} \left(\frac{rJ \varphi}{J} \beta \frac{\partial \varphi}{\partial \eta} \right) + S_{\varphi} + S_{d, \varphi} + S_{c, \varphi} \end{aligned} \quad (1)$$

式中: φ 代表不同变量 u, v, h, m_f 等。 U, V 是计算平面上的速度, α 和 β 是坐标变换系数。 S_{φ} 是方程源项,包括坐标变换附加源项及原方程写成通用形式后的剩余项。 $S_{d, \varphi}$ 是有两相流动时的液滴源项, $S_{c, \varphi}$ 是化学反应源项。

2.2 P ISO 算法迭代方案

(1) 预测步 用时间 t^n 层流动参数计算差分方程的算子系数,变换后的控制方程在计算

平面采用有限容积法离散。预测步控制方程用隐格式离散如下:

$$(\rho^n/\delta t - A_0^n)u_i^* = H(u_i^*) + (\rho/\delta t)u_i^n - \Delta p_i^n + S_{u_i}^n + S_{d,u_i}^n \quad (2)$$

$$(\rho^n/\delta t - B_0^n)h^* = J(h^*) + (\rho/\delta t)h^n - [(p^n - p^{n-1})/\delta t] + S_h^n + S_{d,h}^n \quad (3)$$

$$(\rho^n/\delta t - C_0^n)m_f^* = G(m_f^*) + (\rho/\delta t)m_f^n + S_{m_f}^n + S_{d,m_f}^n + S_{c,m_f}^n \quad (4)$$

其中算子表达式如下:

$$H(u_i^*) = \sum A_{nb}u_{i,nb}^*, \quad J(h^*) = \sum B_{nb}h_{nb}^*, \quad H(m_f^*) = \sum C_{nb}m_{f,nb}^* \quad (5)$$

算子中系数 A, B, C 的下标“0”表示中心节点,下标“nb”表示周围节点,这些系数与所用的差分格式有关。求解(2)~(4)式得到 u_i^*, h^*, m_f^* ,新的温度 T^* 由焓值计算。

(2)第一校正步:经过预测步计算得到的流动参数并不满足连续方程,因此必须进行校正。在预测步时没有计算压力值, PISO 算法也象 SIMPLE 方法一样采用了压力-速度耦合方式,利用预测步离散的连续方程和动量方程推导一个压力方程,用来求解新的压力值。

$$\left[\Delta_i \left(\frac{1}{\delta t} - \frac{1}{RT^*} \frac{1}{\delta t} \right) \right] p_i^* = \Delta_i \left\{ \left[\frac{1}{\delta t} - \frac{A_0^n}{\rho^n} \right]^{-1} \left[H^n(u_i^*) + \frac{\rho^n}{\delta t} u_i^n + S_{u_i}^n + S_{d,u_i}^n \right] \right\} - \frac{\rho^n}{\delta t} \quad (6)$$

这样通过压力方程求解出压力,再由状态方程求出密度 ρ^* 。把 ρ^* 和 p^* 代入下面的动量方程、能量方程、组分方程求出速度 $u_i^{**}, h^{**}, m_f^{**}$,这时的 ρ^* 和 u_i^{**} 是满足连续方程的

$$(\rho^n/\delta t - A_0^n)u_i^{**} = H^n(u_i^{**}) + (\rho/\delta t)u_i^n - \Delta p_i^n + S_{u_i}^n + S_{d,u_i}^n \quad (7)$$

$$(\rho^n/\delta t - B_0^n)h^{**} = J^n(h^{**}) + (\rho/\delta t)h^n - [(p^* - p^n)/\delta t] + S_h^n + S_{d,h}^n \quad (8)$$

$$(\rho^n/\delta t - C_0^n)m_f^{**} = G^n(m_f^{**}) + (\rho/\delta t)m_f^n + S_{m_f}^n + S_{d,m_f}^n + S_{c,m_f}^n \quad (9)$$

(3)第二校正步:为了进一步提高计算精度,又进行了与第一校正步完全类似的第二步校正,只是这时离散方程的系数采用第一校正步求出的参数来计算,而不是用 t^n 时间层的值。第二校正步得到的流场参数 $\rho^{**}, p^{**}, h^{***}, T^{***}, m_f^{***}$ 等则作为 t^{n+1} 时间层的流场参数值。

2.3 二阶迎风格式

实际上所有的控制方程离散后都可写成如下的通用形式:

$$a_p \varphi_p = \sum a_{nb} \varphi_{nb} + a_p^0 \varphi_p^0 + b_p \quad (10)$$

当采用不同差分格式时,得到的周围节点系数 a_{nb} 也不同。对均匀网格,二阶迎风格式为:

$$\frac{\partial \varphi}{\partial x} \Big|_p = \begin{cases} (\varphi_p - \varphi_w)/\Delta x + (1/2\Delta x)(\varphi_p - 2\varphi_w + \varphi_{ww}) & U_p > 0 \\ (\varphi_E - \varphi_p)/\Delta x - (1/2\Delta x)(\varphi_p - 2\varphi_E + \varphi_{EE}) & U_p < 0 \end{cases} \quad (11)$$

式中第一项就是一阶迎风差分,第二项相当于高阶修正项。仿照 SIMPLE 算法中的一阶迎风格式,控制容积边界处流通量即可写成:

$$(\rho u \varphi_e = \varphi_p [F_e, 0] - [\varphi_E - (\varphi_p - 2\varphi_E + \varphi_{EE})/2][|-F_e, 0|] \quad (12)$$

$$(\rho u \varphi_w = [\varphi_p - (\varphi_p - 2\varphi_w + \varphi_{ww})/2][|F_w, 0|] - \varphi_p [|-F_w, 0|] \quad (13)$$

把上面二式代入下式(14),即可得到对流项二阶迎风差分表达式。

$$\alpha \rho u \varphi / \alpha x = (1/\Delta x)[(\rho u \varphi_e - (\rho u \varphi_w)] \quad (14)$$

如果控制方程扩散项应用中心差分离散时,则得到离散方程系数如下,具体推导见参考文献[5]。

$$a_p = a_E + a_w + a_N + a_S + a_{EE} + a_{ww} + a_{NN} + a_{SS} + a_p^0$$

$$\begin{aligned}
 a_E &= D_e + [|-2Fe, 0|] & , & & a_{EE} &= - [|-Fe/2, 0|] \\
 a_W &= D_w + [|2Fw, 0|] & , & & a_{WW} &= - [|Fw/2, 0|] \\
 a_N &= D_n + [|-2Fn, 0|] & , & & a_{NN} &= - [|-Fn/2, 0|] \\
 a_S &= D_s + [|2Fs, 0|] & , & & a_{SS} &= - [|Fs/2, 0|] \\
 a_p^0 &= (\rho_p^0/\delta t) rJ \triangle \xi \triangle \eta & , & & b\varphi &= (S\varphi + S_{d,\varphi}) rJ \triangle \xi \triangle \eta
 \end{aligned}$$

式中符号 $[|a, b|]$ 表示取较大值。

2.4 非交错网格技术

计算中应用非交错网格时需要进行特殊插值处理^[6],即控制容积边界处的速度应用离散的动量方程插值计算,而不是由已计算出的相邻节点速度值直接插值,具体做法如下:

$$u_e = \left[\left(\sum a_{nb} u_{nb} + b_u \right) / a_p \right]_e - \left[\frac{\alpha_p}{a_p} \right]_e (P_E - P_P) \quad (15)$$

式中 α_p 是压力梯度项的离散系数。采用上述插值方法可完全消除由于非交错网格引起的压力振荡。PISO 迭代方案,作者发展的二阶迎风格式和非交错网格技术是算法的三大特点。

3 计算结果及分析

应用上述数值方法对气氢/液氧火箭发动机液雾燃烧流动过程进行了数值模拟计算。采用文献[7]提出的喷雾随机模型确定氧液滴初始条件,化学反应速率应用 Arrhenius 公式计算。图 1 是应用 TTM 方法生成的计算网格,网格线在燃烧室头部、喉部、壁面加密可自由控制。图 2 是推力室的 Mach 数等值线分布,由图可清楚地看到喷管喉部的音速线位置。图 3 是燃气三种组分质量分数沿轴向的分布。氧组分浓度由于液滴在头部区域大量蒸发而增大随后

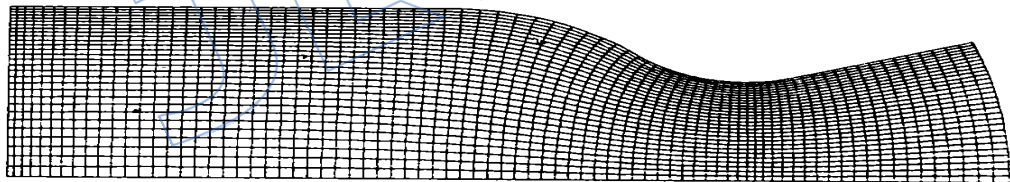


图 1 计算网格

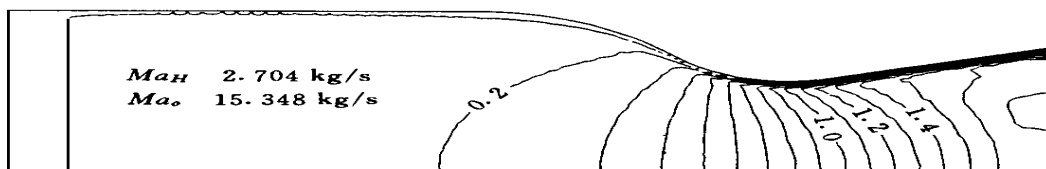


图 2 燃烧室 Mach 数等值线分布图

又因燃烧反应而减小。图 4 是温度沿轴向分布曲线,温度在开始段迅速上升到平衡温度,表明氢氧化学反应在靠近燃烧室头部很近的区域内就完成了。图中三角符号是应用试验验证过的热力计算程序得到的结果,作为参考比较。

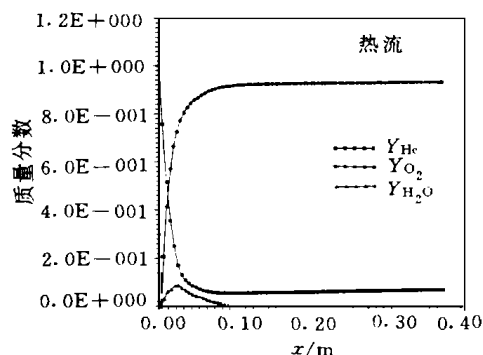


图3 燃气组分质量分数沿轴向分布

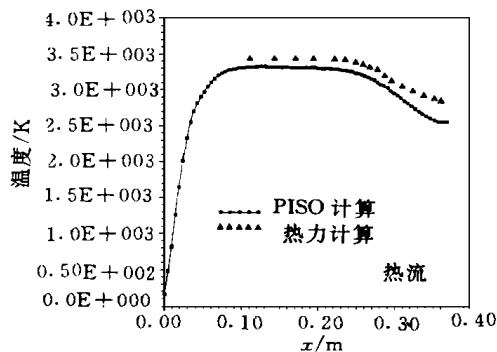


图4 燃气温度沿轴向分布曲线

通过数值计算表明本文发展的二阶迎风格式与 PISO 算法的结合是成功的, 完全可以用于发动机燃烧过程数值模拟计算。

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that the film cooling effectiveness would be much greater than that found in the downstream of a circular hole

Key Words Jet Flow fields Experiment

A SURVEY ON PROPELLANT LEAK DETECTION TECHNIQUES FOR LIQUID ROCKET PROPULSION SYSTEM

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ABSTRACT The integrity test of Liquid Rocket Engine (LRE) must be carried out before its launch. The tests include the leak detection of liquid propellant and other liquid systems. It is necessary to set up an automated propellant leak detection system. This paper summarizes the leak detection techniques of LRE, which are classified into groups and considered in details. Finally, several prospects of the leak detection techniques are presented.

Key Words Liquid rocket engines Propellants Leakage Fault detection

“POWER- LAW ”AND “QUICK ”SCHEMES IN NUMERICAL STUDY OF NONSYMMETRY DIFFUSER

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ABSTRACT Speziale's nonlinear k - ϵ model is used to calculate the turbulent flow in nonsymmetry diffuser. Attention is focused on effect of the two typical convective difference schemes, “Power- Law ” and “QUICK ” schemes, on the performance of the models (the standard and nonlinear k - ϵ models). The results show that both convective difference schemes have little effect on the prediction of the models. So it is no longer to puzzle brain about choice of convective difference schemes, when applying to nonlinear k - ϵ models.

Key Words Diffusers Turbulent flow Mathematical models
Numerical computation

APPLICATION OF A SECOND- ORDER UPWIND SCHEME TO PISO ALGORITHM

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ABSTRACT Numerical simulation of spray combustion in liquid propellant rocket engine with PISO algorithm was implemented. A second- order upwind scheme developed by the authors was combined with the PISO algorithm to ensure the numerical accuracy. The Navier- Stokes equation in cylindrical coordinates was transferred into general curvilinear coordinate system and discretized by finite- volume method with non- staggered grid. The body- fitted grid generated by TTM method was used in the calculation. A stochastic spray model was employed to determine the drop let initial conditions, and the chemical reac-

tion rate was given by Arrhenius model. The results show that the numerical method and physical models are proper for simulation of spray combustion processes.

Key Words Spray combustion flow Calculation Algorithm

AN EXPERIMENTAL STUDY ON PRIMARY NOZZLE TYPES OF EXHAUST EJECTOR SYSTEMS

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ABSTRACT Optimum distances between primary nozzle exit and mixing tube inlet, static pressure recovery of the mixing flow in mixing tube, total pressure distributions at the exit of mixing tube, pumping coefficients and total pressure losses of the exhaust ejector systems with four different types of primary nozzles have been measured in the experiments. The results have an important reference value for selecting primary nozzle types of exhaust ejector systems.

Key Words Nozzle with injector Aerodynamic tests Experiments

TECHNICAL NOTES

NONLINEAR PERTURBATION METHOD FOR ANALYSIS OF MODAL SENSITIVITIES WITH REPEATED EIGENVALUES

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ABSTRACT An algorithm was made out for analyzing the modal sensitivities with repeated eigenvalues of real symmetric matrices by means of nonlinear perturbation equation. It has advantage in retaining the symmetry and band structure of the original eigen-system matrices. A numerical example shows that the method has high accuracy and can be easily implemented on computer.

Key Words Modal analysis Eigenvalues Perturbation method

TRANSIENT RESPONSES OF BLADES TO BIRD IMPACT COUPLED WITH DEFORMATIONS OF BLADE

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ABSTRACT It is necessary to calculate the transient responses of blades precisely in analysis of blade damage due to bird impact. Because the deformations of blade can